Subsonic and Transonic Potential Flow over Helicopter Rotor Blades

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Compressible potential flows over nonlifting, hovering helicopter blades are described by suitable linear and nonlinear equations of motion for subsonic and transonic cases, respectively. Analytical and numerical results are presented for the linearized subsonic three-dimensional flow in the tip region. When the tip Mach number is transonic, the flowfield is calculated using a computational method that is a formal extension to three dimensions of recently developed nonlinear two-dimensional relaxation schemes. Calculations are presented for rectangular blades with 6% thick biconvex sections. Calculations show the relative importance of tip Mach number and aspect ratio on the growth and extent of shock waves in the tip region, and indicate a significant reduction in shock strength with decreasing aspect ratio.

Introduction

HELICOPTER rotor blades that can operate at high tip Mach numbers without the penalties usually associated with super-critical flow would permit high-forward speeds at relatively low-advance ratios. As the advance ratio decreases, the portion of the retreating blade that is stalled would be diminished, the portion of the retreating blade that is unstalled could operate at smaller angles of attack, and the advancing blade could generate lift without also generating an excessive rolling moment. Vibratory loads would be diminished, and severe unsteady pitching moments on the lifting portion of the retreating blade would decrease if that portion could usefully operate at higher tip speeds and smaller angles of attack.

The design of rotor blade geometries that can support a transonic flowfield that is nearly shock-free requires a method for computing the detailed pressure distribution on the blade for a specified rotor geometry and advance ratio. However, the unsteady aerodynamic environment of a lifting rotor that is operating at or near the critical Mach number is extremely complicated. It is easy to appreciate the difficulty of describing such flowfields when one recalls that we do not yet have an adequate model for describing the details of the flow about a lifting, hovering rotor operating in an essentially incompressible flow. Our purpose here is therefore to focus on a portion of the high-speed rotor problem that is sufficiently limited to allow for some analytical and numerical treatment, but which retains important linear and nonlinear features of the flow. We consider a two-bladed nonlifting and hovering rotor operating in the range of tip Mach numbers where compressibility effects are important. The pressure distribution near the rotor tip is studied. In this region, the flowfield is fully three-dimensional, and blade element (strip) theory is in no sense a reasonable approximation to the rotor aerodynamics. It is assumed that the flow is inviscid. The flow is steady relative to a coordinate system attached to the blades. Analytical and numerical results presented here are based on suitable approximations to the full three-dimensional potential equation for compressible flow. When the tip Mach number is subcritical, the potential equation is the usual acoustic equation in an inertial coordinate system, and is a more complicated but still linear equation when it is rewritten in the rotating coordinate system. For tip Mach numbers near or above the critical Mach number, the flowfield in the region of the tip is described by a potential equation that retains the appropriate nonlinearity to account for transonic effects.

In the subcritical case, the solution to the linearized potential equation that is valid in the neighborhood of the tip can be determined as an asymptotic series in inverse powers of the blade aspect ratio. Near the tip of any large aspect ratio fixed wing or rotor blade the three-dimensional flowfield is, to a first approximation, independent of aspect ratio. This result is based on the fact that the characteristic length near the tip is the blade chord, and therefore, in the first-order potential solution the blade appears to be semi-infinite as viewed from the tip. The next term in the solution is proportional to AR^{-1}, and represents the first effect on the flowfield due to the finite span of the blade and the finite radius of curvature of streamlines near the tip. The asymptotic solution for the subcritical problem can be determined by the method of matched asymptotic expansions. A simpler but equivalent procedure is used here to find a uniformly valid first approximation that gives the correct three-dimensional flow near the tip, and which reduces to blade element theory inboard from the tip. This simple composite solution shows in one formula the competition between the effect of increasing effective Mach number as the tip is approached, and the tendency of the flow to become more three-dimensional near the tip and thereby provide some relief from compressibility effects.

When conditions at the tip are transonic, the nonlinear potential equation must be numerically integrated. The numerical methods used here differ from those of Ref. 1 in that we do not use superposition of retarded potentials to represent blade thickness. Relaxation methods, used here, appear to be a simpler method for this linearized three-dimensional problem and are essential for the nonlinear transonic problem.

The computational method is a formal extension to three dimensions of the nonlinear relaxation schemes which have recently been developed.[2,3] A point relaxation scheme is used. This method was also used for the usual three-dimensional equation for a nonrotating wing in order to compare the tip effects. For the rotor case, machine storage necessitates beginning the calculation as far out on the blade as possible, but far enough from the tip that three-dimensional effects are negligible. At this span station a precalculated two-dimensional problem solution is used for a boundary condition. The number of iterations required for a good solution are of the same order as for two-dimensional calculations, total running time being about one hour on an IBM 360-67.

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Formulation of the Problem

Equations of Motion

It is assumed that the rotor is hovering and nonlifting, and that the flowfield is inviscid. The flow relative to a coordinate system attached to the rotating blades therefore appears to be a steady one. The blades rotate in the (x, y) plane with constant angular velocity $\boldsymbol{\Omega}$ directed along the z-axis. The velocity vector $\boldsymbol{V}$ of a particle relative to an observer in the rotating coordinate system is

$$\boldsymbol{V} = -\boldsymbol{\Omega} x \boldsymbol{r} + \nabla\phi \tag{1}$$

where $\boldsymbol{r}$ is the position vector in the rotating system, ∇ is the gradient operator, and ϕ is the velocity potential of the disturbance caused by the blades. The complete nonlinear compressible potential equation for ϕ, written in the rotating coordinate system, is

$$\boldsymbol{\Omega} x\boldsymbol{r}\cdot\nabla(\boldsymbol{\Omega} x\boldsymbol{r}\cdot\nabla\phi) - 2\nabla\phi\cdot\nabla(\boldsymbol{\Omega} x\boldsymbol{r}\cdot\nabla\phi) + \nabla\phi\cdot\nabla[\tfrac{1}{2}(\nabla\phi)^2] = a^2\nabla^2\phi \tag{2}$$

where a is the local sound speed. It is related to ϕ through Bernoulli's equation

$$a^2 = a_\infty^2 - (\gamma - 1)[-(\boldsymbol{\Omega} x\boldsymbol{r})\cdot\nabla\phi + \tfrac{1}{2}(\nabla\phi)^2]$$

where γ is the ratio of specific heats and a_∞ is the sound speed in the rest medium.

Boundary Conditions

The mean surface of the (thin) rotor blade lies in the (x, y) plane (see Fig. 1). Cartesian (x, y, z) coordinates are used, with x in the blade chord direction, y along the blade span axis and z normal to the blade planform. The blade geometry is described by

$$F(x, y, z) \equiv z - f(x, y) = 0$$

The condition of flow tangency on the blade surface is transferred to the blade mean surface $z = 0$. This tangency condition is

$$(-\boldsymbol{\Omega} x\boldsymbol{r} + \nabla\phi)\cdot\nabla F = 0 \quad \text{on } z = 0 \tag{3}$$

Letting $\boldsymbol{\Omega} = \Omega\boldsymbol{k}$, with $\boldsymbol{k}$ the unit vector along the z axis, Eq. (3) is

$$\phi_z = (\Omega y + \phi_x)f_x + (-\Omega x + \phi_y)f_y \qquad (z = 0) \tag{4}$$

where subscripts indicate partial differentiation. Equation (4) is the boundary condition on the blade for a hovering rotor; it is the appropriate tangency condition whether or not the blade is lifting.

It follows from Eq. (1) that $\nabla\phi$ is a perturbation potential, for if the blades were not present we would have $\nabla\phi = 0$ and $\boldsymbol{V} = -\boldsymbol{\Omega} x\boldsymbol{r}$, which, to the observer in the rotating system, is a rotary clockwise motion about the z axis. Hence one boundary condition is that $\nabla\phi \to 0$ far from the blade.

With y the coordinate along the span axis and ϕ a perturbation potential, we should have $\Omega y \gg \phi_x$ away from the center of rotation. If there is no thickness taper, then $f_y = 0$. Taper in blade thickness could be included and the second group of terms on the right-hand side of Eq. (4) would be much smaller than the first group, as long as the thickness taper is reasonably gentle. Thus the condition of flow tangency becomes approximately

$$\phi_z(x, y, 0) = \Omega y f_x(x, y) \tag{5}$$

The condition at infinity is

$$\nabla\phi \to 0 \text{ as } x^2 + y^2 + z^2 \to \infty$$

The Potential Equation

Since each point on the blade executes a circular motion, and since the streamlines are nearly circular in the blade tip region, it is tempting to describe the flow in a cylindrical coordinate system, with polar (r, θ) variables in the plane of the blade motion. However, in the tip region, the radius of curvature of the streamlines is so large relative to the blade chord dimension (for large aspect ratio blades) that an observer moving with the blade tip sees a nearly rectilinear motion of the fluid particles in that region. It should therefore be convenient to describe the motion in the tip region in a Cartesian coordinate system to take advantage of this nearly rectilinear motion. The foregoing statements are merely a way of stating the basic assumptions of blade element or strip theory. These statements remain correct when the essentially three-dimensional motion near the tip is considered.

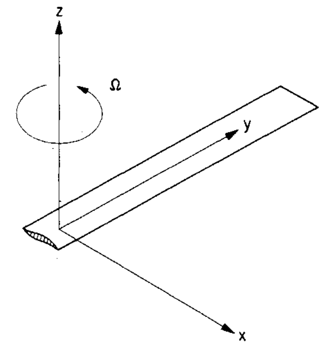

Fig. 1 Rotating coordinate system.

When the basic differential Eq. (2) is written out in scalar form, and a^2 is substituted from Bernoulli's equation, the result is

$$\begin{aligned}&\{(\Omega y)^2 - a_\infty^2 + (\gamma + 1)\Omega y\phi_x - (\gamma - 1)\Omega x\phi_y + \phi_x^2 \\ &+ [(\gamma - 1)/2](\phi_x^2 + \phi_y^2)\}\phi_{xx} + \{(\Omega x)^2 - a_\infty^2 - (\gamma + 1)\Omega x\phi_y \\ &+ (\gamma - 1)\Omega y\phi_x + \phi_y^2 + [(\gamma - 1)/2](\phi_x^2 + \phi_y^2)\}\phi_{yy} \\ &+ \{-a_\infty^2 + (\gamma - 1)\Omega y\phi_x - (\gamma - 1)\Omega x\phi_y + \phi_z^2 \\ &+ [(\gamma - 1)/2](\phi_x^2 + \phi_y^2)\}\phi_{zz} - [2\Omega^2 xy + 2\Omega x\phi_x \\ &- 2\Omega y\phi_y - 2\phi_x\phi_x]\phi_{xy} + 2\Omega\phi_z(y\phi_{xz} - x\phi_{yz}) + 2\phi_x\phi_z\phi_{xz} \\ &+ 2\phi_y\phi_z\phi_{yz} - \Omega^2 x\phi_x - \Omega^2 y\phi_y = 0\end{aligned} \tag{6}$$

Special Cases

There are three useful specializations of Eq. (6).

Incompressible flow

In Eq. (6), divide by a_∞^2 and let $a_\infty \to \infty$. Equation (6) reduces to

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \tag{7}$$

Linearized Subsonic or Supersonic Flow

Retain only linear terms in Eq. (6). The result is, after some rearrangement,

$$[a_\infty^2 - (\Omega y)^2]\phi_{xx} + [a_\infty^2 - (\Omega x)^2]\phi_{yy} + a_\infty^2\phi_{zz} + 2\Omega^2 xy\phi_{xy} + \Omega^2 x\phi_x + \Omega^2 y\phi_y = 0 \tag{8}$$

Nonlinear Transonic Flow

Let R be the blade radius. Transonic flow means that the blade tip velocity ΩR is close to the sound speed a_∞ in the rest medium, or that $\Omega y \sim a_\infty$ when y is near R. Proceeding by analogy with the derivation of the nonlinear transonic equation for a fixed wing, Eq. (6) specializes to the transonic rotor equation

$$[a_\infty^2 - (\Omega y)^2 - (\gamma + 1)\Omega y\phi_x]\phi_{xx} + a_\infty^2\phi_{yy} + a_\infty^2\phi_{zz} + 2\Omega^2 xy\phi_{xy} + \Omega^2 x\phi_x + \Omega^2 y\phi_y = 0 \tag{9}$$

This equation includes the appropriate nonlinearity and describes the region of transonic flow near the blade tip.

Scaling of Equations

More information about the flowfield is obtained when the dependent and independent variables are made dimensionaless and are properly scaled. The blade geometry will be restricted to a family of rectangular blades of chord c and radius R, with blade sections affinely related by

$$z = c\tau g(x/c) \tag{10}$$

where τ is the thickness ratio and g is assumed to be of order unity.

The variables ϕ, x, y, and z in Eqs. (7–9) are dimensional. These variables are to be replaced by dimensionaless scaled variables. To avoid an undue proliferation of symbols, dimensional

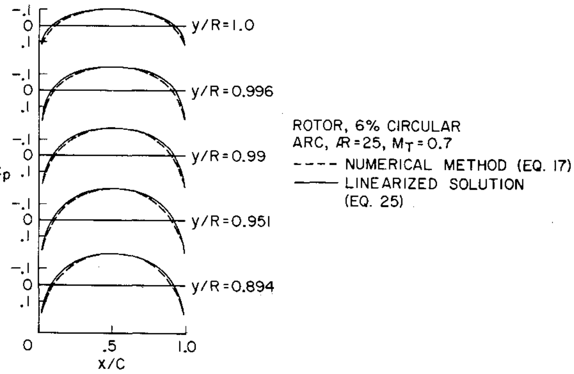

Fig. 2 Comparison of numerical and analytic results.

variables ϕ, x, y, and z will be replaced by new dimensionaless scaled variables by the substitutions

$$\phi \to \delta\Omega Rc\phi, \qquad x \to cx, \qquad y \to Ry, \qquad z \to c\mu z \tag{11}$$

where this notation indicates that the quantities ϕ, x, y, and z that appear in Eqs. (7–9) are to be replaced by the right-hand sides of the above relations. The constants δ and μ in Eq. (11) are scale factors to be determined.

For blade geometry defined by Eq. (10) and the notation of Eq. (11), the tangency condition, Eq. (5), becomes

$$\phi_z = (\tau/\mu\delta)y\dot{g}(x) \tag{12}$$

The blade span axis is placed along the blade midchord line. The blade geometry from leading edge to trailing edge and tip to tip is defined by

$$-\tfrac{1}{2} \leqq x \leqq \tfrac{1}{2}, \qquad -1 \leqq y \leqq 1$$

The right-hand side of Eq. (12) is of order unity if μ is defined by

$$\mu = \tau/\delta \tag{13}$$

Two additional important parameters are the blade tip Mach number M_T and the inverse of the blade aspect ratio ε

$$M_T = \Omega R/a_\infty, \qquad \varepsilon = 1/AR = c/R \tag{14}$$

Using the substitutions required by Eq. (11) in Eqs. (7–9) and the definitions Eqs. (13) and (14), the three distinguished equations of motion (7–9), become

Incompressible flow

Let the scale factor $\delta = \tau$. Equation (7) becomes

$$\phi_{xx} + \varepsilon^2\phi_{yy} + \phi_{zz} = 0$$

with boundary condition

$$\phi_z = y\dot{g}(x) \quad \text{on } z = 0$$

Blade element theory is based on setting $\varepsilon = 0$ and noting that the spanwise coordinate y becomes only a parameter in the problem.

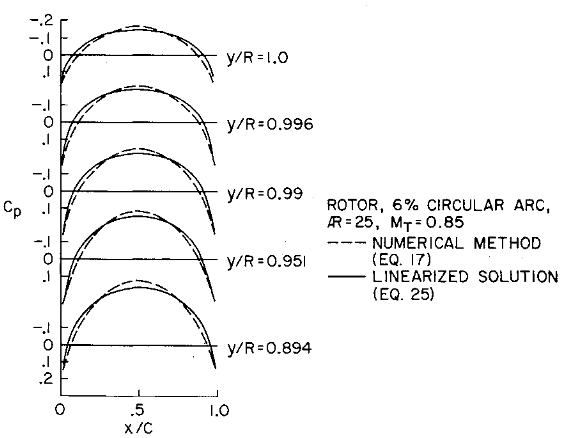

Fig. 3 Comparison of numerical and analytic results.

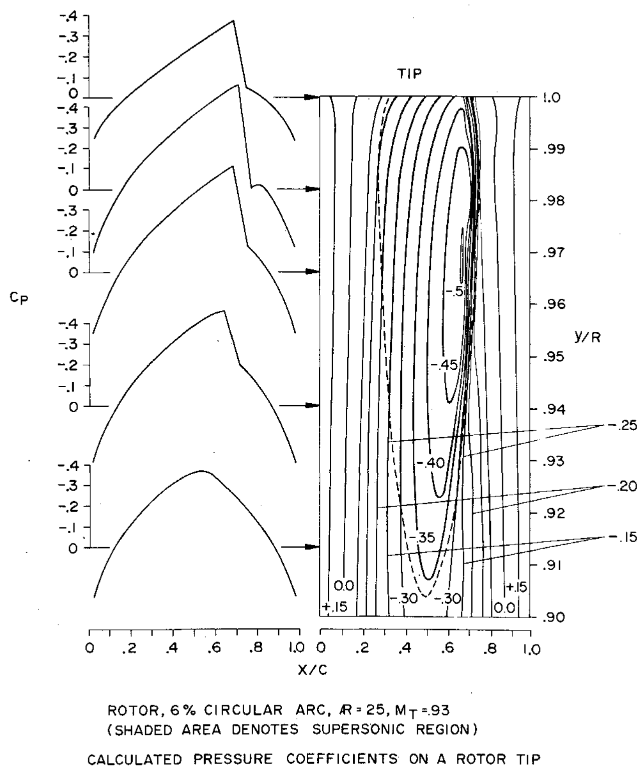

Fig. 4 Calculated pressure coefficients on a rotor tip.

Linearized subsonic or supersonic flow

Again let $\delta = \tau$. Equation (8) becomes

$$(1 - M_T^2y^2)\phi_{xx} + \varepsilon^2\phi_{yy} + \phi_{zz} + M_T^2\varepsilon^2(2xy\phi_{xy} + x\phi_x + y\phi_y) = 0 \tag{15}$$

A blade element theory that includes compressibility effects is based on setting $\varepsilon = 0$, which gives

$$(1 - M_T^2y^2)\phi_{xx} + \phi_{zz} = 0 \tag{16}$$

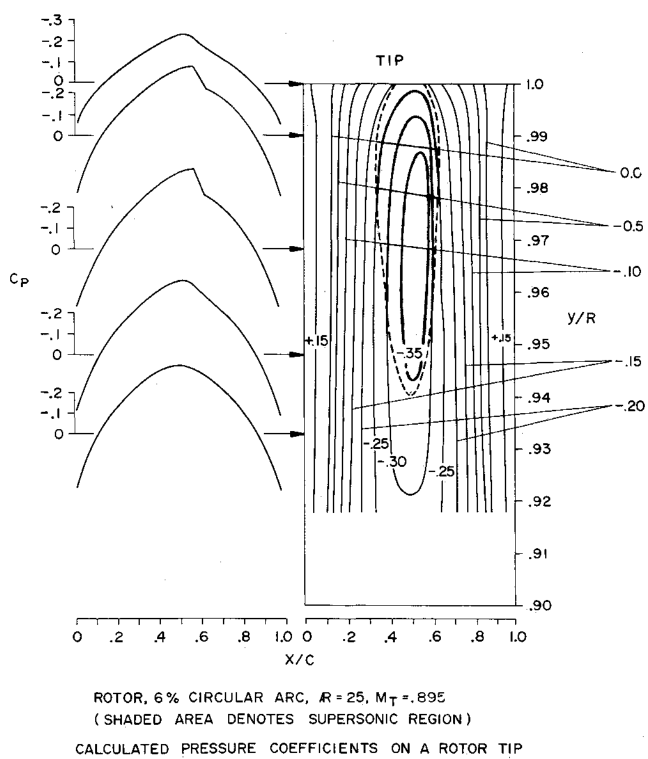

Fig. 5 Calculated pressure coefficients on a rotor tip.

with boundary condition

$$\phi_z = y\dot{g}(x) \quad \text{on } z = 0$$

The spanwise variable y again appears only as a parameter.

Transonic flow

The scale factor δ in Eq. (11) is defined to be

$$\delta = \tau^{2/3}/[(\gamma + 1)M_T^2]^{1/3}$$

Equation (9) becomes

$$\left\{\frac{1 - M_T^2 y^2}{[(\gamma + 1)M_T^2\tau]^{2/3}} - y\phi_x\right\}\phi_{xx} + \frac{\varepsilon^2}{[(\gamma + 1)M_T^2\tau]^{2/3}}\phi_{yy} + \phi_{zz} + \frac{M_T^2\varepsilon^2}{[(\gamma + 1)M_T^2\tau]^{2/3}}[2xy\phi_{xy} + x\phi_x + y\phi_y] = 0 \qquad (17)$$

with boundary condition

$$\phi_z = y\dot{g}(x) \quad \text{on } z = 0$$

Transonic conditions for a rotor blade are defined to be the conditions that make the term

$$K \equiv (1 - M_T^2 y^2)/[(\gamma + 1)M_T^2\tau]^{2/3}$$

to be of order unity. When M_T is near one and y is near the tip ($y \sim 1$), the values of thickness ratio τ that make K to be of order unity also make the term $y\phi_x$ in Eq. (17) (which produces the nonlinearity) to be of order unity.

Although Eq. (17) is designed to account for three-dimensional nonlinear effects of transonic flow in the tip region, it is valid over the entire blade span $-1 \leqq y \leqq 1$. As y decreases inboard from the tip $K(y)$ becomes large and the nonlinear factor $y\phi_x$ becomes less important. There is, therefore, some station inboard from the tip where the flow ceases to be transonic in nature, but where compressibility effects are still important. Still further inboard, compressibility effects become altogether negligible and the flow becomes essentially incompressible.

Blade element theory for incompressible and linearized compressible flow requires $\varepsilon^2 = AR^{-2}$ to be small. Inspection of Eq. (17) shows that a transonic blade element theory would require the more restrictive assumption $\varepsilon^2\tau^{-2/3} \ll 1$. For a blade of aspect ratio 20 with 12% thick sections, $\varepsilon^2\tau^{-2/3} \sim 0.01$, and the above requirement is certainly met. However, any blade element theory is ultimately based on the physical condition that the flow be nearly two-dimensional. When a rotor tip Mach number is near 1, transonic conditions appear to exist only over the last chord or two of the tip region. In this region close to the tip, the flow is essentially three-dimensional, and the simplifying concept of a transonic blade element theory has a very limited utility, at least for rectangular blades that are not tapered in thickness.

Linearized Tip Solution

Near the blade tip the characteristic dimension is the blade chord. The appropriate spanwise variable in the tip region is therefore

$$s = (1 - y)/\varepsilon$$

so that s is distance from the tip, measured in chords. An effective Mach number M_e at each spanwise station is

$$M_e = M_T y = M_T(1 - \varepsilon s)$$

The Mach number gradient along the span axis is

$$dM_e/ds = -\varepsilon M_T$$

which is generally a small quantity for a rotor blade. For a blade of infinite aspect ratio, $\varepsilon = 0$ and the Mach number gradient measured in chords is zero. Hence, the flow in the tip region of a large aspect ratio blade is, to a first approximation ($\varepsilon = 0$), exactly the same as the three-dimensional flow in the tip region of a semi-infinite fixed wing moving at the blade tip Mach number M_T. This three-dimensional flow near the tip can be

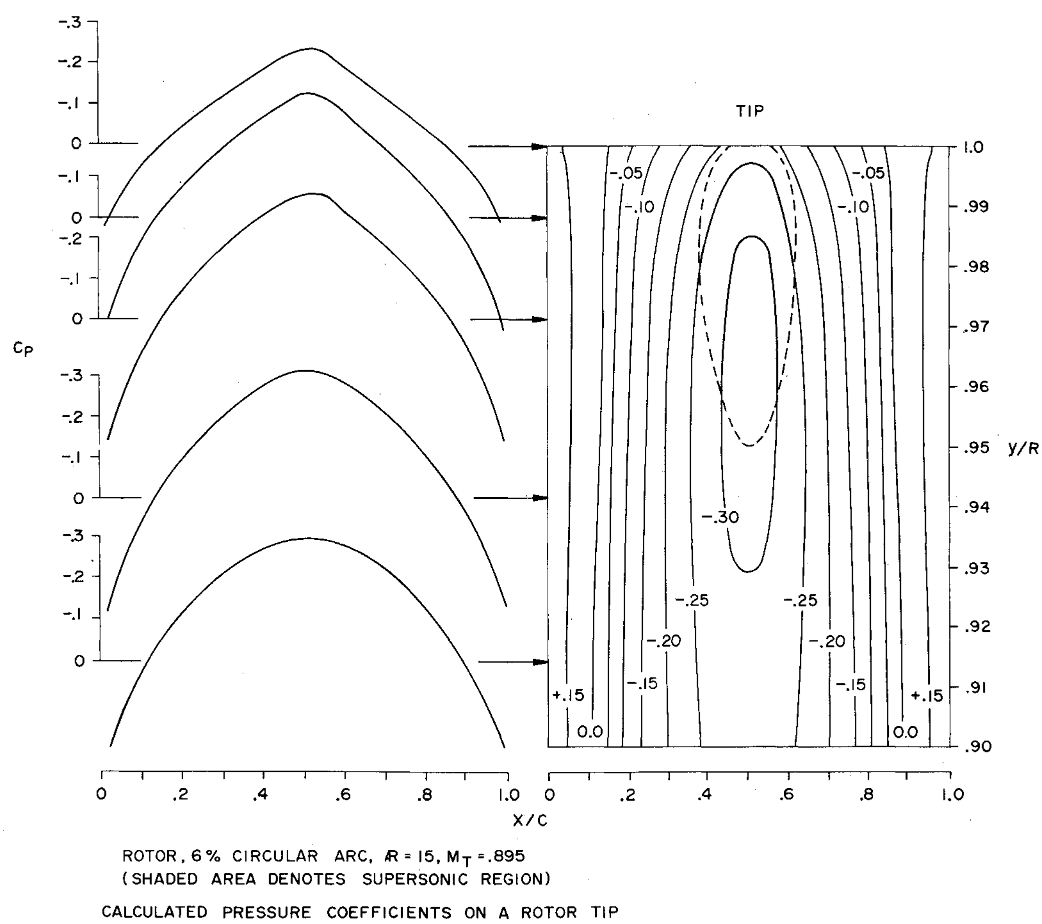

Fig. 6 Calculated pressure coefficients on a rotor tip.

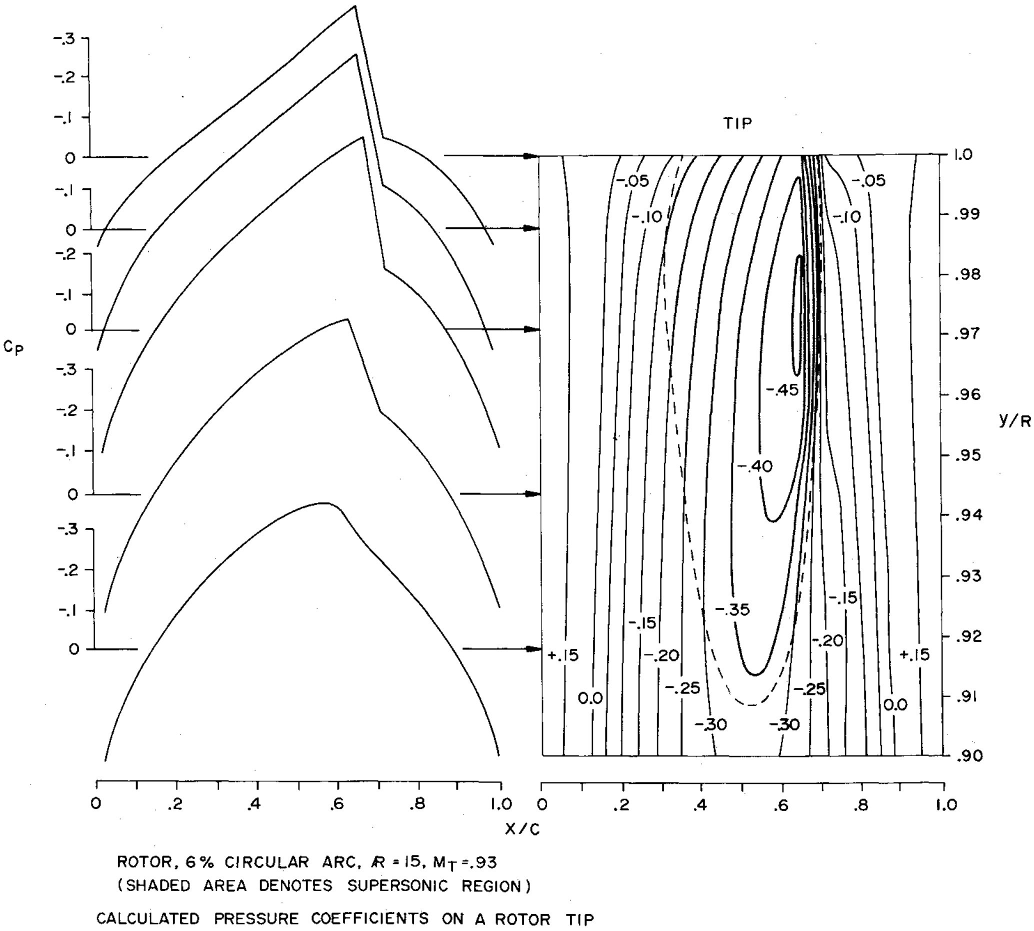

Fig. 7 Calculated pressure coefficients on a rotor tip.

determined by rewriting the differential Eq. (15) for linearized compressible flow in terms of s rather than y, and retaining only terms independent of ε. The result is

$$\beta^2\phi_{xx} + \phi_{ss} + \phi_{zz} = 0 \tag{18}$$

where

$$\beta^2 = 1 - M_T^2$$

and the tangency condition is

$$\phi_z = \dot{g}(x) \quad \text{on } z = 0 \tag{19}$$

The solution to the differential Eq. (17) with boundary condition (19) is easily found. The differential equation is first reduced to Laplace's equation by a Prandtl-Glauert transformation

$$S = \beta s, \qquad Z = \beta z, \qquad \Phi = \beta\phi \tag{20}$$

The problem of solving Eqs. (18) and (19) is transformed to solving

$$\Phi_{xx} + \Phi_{SS} + \Phi_{ZZ} = 0; \qquad \Phi_Z = \dot{g}(x) \quad \text{on } Z = 0 \tag{21}$$

What is now required is the three-dimensional solution for the potential Φ in the tip region of a fixed wing in incompressible flow, such flow being described by Eq. (21). This incompressible tip solution is determined by finding the exact potential Φ (within the mean-surface approximation) for the entire flowfield on a fixed wing of aspect ratio ε^{-1}, expressing this solution in tip variables x, s, and z, letting $\varepsilon \to 0$, retaining only the leading term in the expression for Φ, and then setting $\phi = \Phi/\beta$ according to Eq. (20). The result is

$$\phi \sim (1/\beta)[F(x, \beta z) + G(x, \beta s, \beta z)] \tag{22}$$

where the functions F and G are

$$F = \frac{1}{2\pi}\int_{-1/2}^{1/2} \dot{g}(\xi)\log\left[(x-\xi)^2 + \beta^2 z^2\right] d\xi$$

$$G = -\frac{1}{2\pi}\int_{-1/2}^{1/2} \dot{g}(\xi)\log\left\{\beta s + \left[(x-\xi)^2 + \beta^2 s^2 + \beta^2 z^2\right]^{1/2}\right\} d\xi$$

For large s, far inboard, the function G approaches zero and

$$\phi \sim \frac{1}{\beta}\cdot\frac{1}{2\pi}\int_{-1/2}^{1/2} \dot{g}(\xi)\log\left[(x-\xi)^2 + \beta^2 z^2\right] d\xi (s \to \infty) \tag{23}$$

Far inboard, the rotor potential ϕ should approach the blade element result determined by Eq. (16). That solution is

$$\phi \sim \frac{y}{(1 - M_T^2 y^2)^{1/2}}\frac{1}{2\pi}\int_{-1/2}^{1/2} \dot{g}(\xi)\log\left[(x-\xi)^2 + (1 - M_T^2 y^2)z^2\right] d\xi \tag{24}$$

which is not the same as the ϕ given by Eq. (23). However, to the approximation considered here, a solution for the rotor potential ϕ that is uniformly valid over the entire blade $0 \leqq y \leqq 1$ is

$$\phi \sim \frac{y}{(1 - M_T^2 y^2)^{1/2}}\left[F(x, 0) + G(x, \beta s, 0)\right] \tag{25}$$

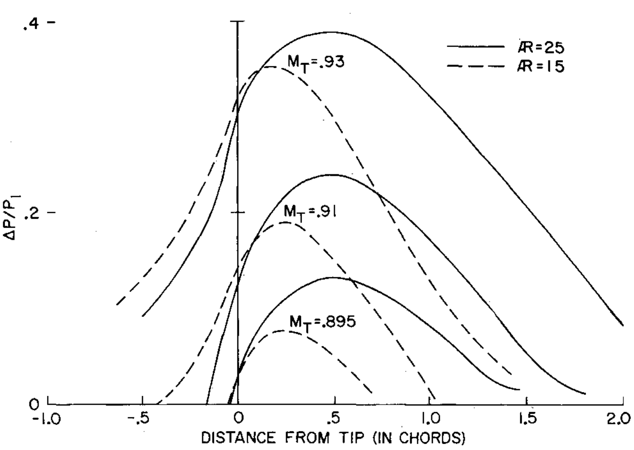

Fig. 8 Effect of tip Mach number and aspect ratio on shock strength.

The solution ϕ in mixed variables s and y given by Eq. (25) becomes the three-dimensional tip solution (22) when $y \sim 1$, and reduces to the blade element result (24) with $z = 0$ when s is large. The error in Eq. (25) is $0(\varepsilon)$ near the tip and $0(\varepsilon^2)$ at large distance s inboard from the tip.

The solution (25) has been derived here by elementary methods. The solutions (22) or (25) are simply first terms in an inner asymptotic expansion, in powers of ε, of the solution ϕ to the complete rotor Eq. (15), when that equation is rewritten in terms of the tip variable s rather than the more conventional variable y. Such an inner solution must be matched with an outer solution for the potential of a system of compressible doublets distributed along the span axis y and rotating in the (x, y) plane about the z axis, the doublets being derived from retarded source-sink potentials. Details of this method for describing linearized compressible flow on a rotor blade will be described elsewhere.

Pressure Coefficient

Throughout the calculations, the formula for pressure coefficient is

$$C_P = -(2\phi_x/y)$$

where ϕ and y are the dimensionaless potential and spanwise variable defined by the relations (11).

Numerical Method

Equation (17) has been numerically treated using a 7-point computational molecule, the chordwise derivatives being simulated by central or backward differences according to whether the local flow is subsonic or supersonic. Since uniform mesh size is used in the normal and chordwise directions (on the body), the corresponding differences are second order in the subsonic flow region. The backward chordwise differences used in the supersonic region are first order accurate. For large aspect ratios, the sudden tip relief necessitates considerable mesh refinement. It was necessary to use a spanwise mesh size of about 0.025 chords at the tip. Because of the variable spanwise mesh size the corresponding differences are of first order.

The resulting set of nonlinear algebraic equations are solved by successive over relaxation. Experiments with two-dimensional calculations have shown this method to be very efficient up to the lower transonic range. Above this range the convergence rate decreases markedly and line relaxation might become more advantageous. Two relaxation factors are used simultaneously, 1.8–1.9 in the subsonic regions and 0.8–0.9 in the supersonic regions. The relaxation rate for three-dimensional calculations is very similar to that of a comparable two-dimensional problem. Calculations for a rotor with a 6% circular arc profile using a $64 \times 30 \times 30$ mesh will have adequately converged within 75–170 iterations for Mach numbers not exceeding about 0.91. Between $M_T = 0.91$ and $M_T = 0.93$ the running time trebles. This is quite serious, as an average running time on an IBM 360/67 is about an hour.

To economize on machine storage we compute only a small portion of the flow near the tip. A previously calculated strip theory result is specified at a span station sufficiently inward to be free of any tip influence and the solution is carried on from this point. A distance of two chords seems sufficient up to low-transonic speeds. Four chords were used for higher transonic speeds. On the outer boundaries the potential is set to zero. These boundaries are about $1\frac{1}{2}$ chords removed from the rotor in the chordwise direction, two chords removed spanwise and five chords normal to the rotor.

Discussion of Results

All the results to date are for rectangular rotors with a 6% circular profile whose axis of rotation lies on the midchord line. This is a convenient profile because calculations converge rapidly. Furthermore, having fore and aft symmetry and a centered axis of rotation, all departures of the flow from symmetry are immediately recognized as due to the presence of local regions of supersonic flow.

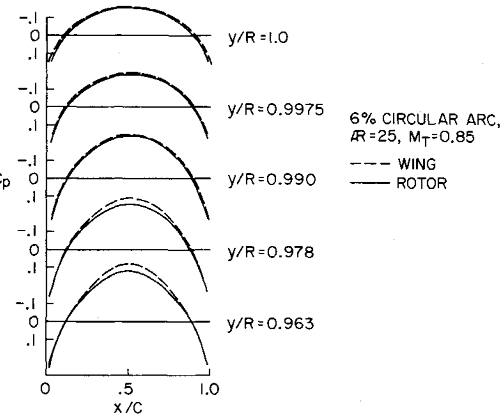

Fig. 9 Comparison of tip solution for a wing and rotor.

To check the validity of our calculations, they are compared to the analytical solution, Eq. (25), which was integrated in closed form for biconvex sections. Figure 2 shows a comparison of numerically and analytically derived pressure coefficients at several span stations for a tip Mach number of 0.7. A similar comparison (Fig. 3) made for a tip Mach number of 0.85—a completely subsonic case—shows the linear theory to be rather poor over the entire tip region. This indicates that with the onset of transonic flow, the tip relief, though sometimes sufficient to render the tip flow subsonic, cannot be so great as to render valid any linear theory at the tip.

In order to assess some of the factors that affect the competition between tip relief and increasing M_e, a series of calculations at various aspect ratios and tip Mach numbers was made. The influence of the tip is mainly felt for a distance of about a chord (this distance increasing, of course, as M_T approaches 1). Since for decreasing aspect ratio, this distance encompasses a greater percentage of the blade, we expect a less expanded flow in the over-all tip region. Other indications of the relative influence of tip relief and increasing M_e are the spanwise locations of the point of minimum C_p, and maximum local Mach number and shock strength. For the biconvex profile the point of maximum shock strength coincides with the maximum local Mach number, if a shock does occur. Furthermore, because of the increasing M_e the maximum shock occurs closer to the tip than the minimum C_p location. In Figs. 4–7 it is seen that for any given M_T the blades of aspect ratio 15 have lower C_p's and smoother flows than the blades of aspect ratio 25. In fact, for M_T of .895, Figs. 5 and 6, both flows are transonic, but the lower aspect ratio has no readily discernible shocks at all. Note that for a given aspect ratio the point at which minimum C_P occurs is unaffected by

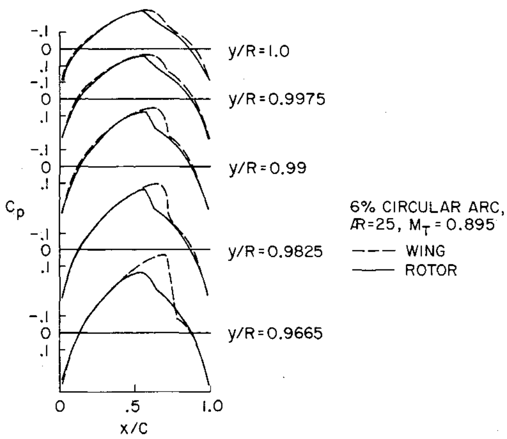

Fig. 10 Comparison of tip solution for a wing and rotor.

changing M_T. This phenomenon is best seen in Fig. 8, in which the six flows are summarized by a comparison of shock strength, calculated on the basis of the maximum local M_E, using the Rankine-Hugoniot relations. It is seen here that the blade of aspect ratio 15 is a considerable improvement over the blade of aspect ratio 25. The point of maximum shock strength is farthest from the tip for the larger aspect ratio. This is to be expected, as increasing aspect ratio decreases the apparent Mach number gradient in the tip coordinate system. An infinite aspect ratio rotor would look like a fixed wing in this system, in which case minimum C_P would be infinitely far from the tip.

In the immediate vicinity of the tip, the flows for a wing with freestream Mach number M_∞ and for a rotor with tip Mach number $M_T = M_\infty$ should be nearly identical, when the wing and rotor aspect ratios are the same. Calculations to check this result are shown in Figs. 9 and 10. When no shock occurs, as in Fig. 9, the agreement at the tip is quite good. However, as M_∞ and M_T increase, the nonlinear behavior of the wing flow near the tip becomes more severe than that of the rotor. This difference in flow behavior must be attributed to the effective Mach number gradient along the rotor span, which restricts nonlinear effects to a very small region near the tip, whereas such nonlinear behavior exists along the entire span of the wing.

Conclusions

Three-dimensional flowfields with embedded supersonic regions have been calculated for rectangular rotor blades with 6% thick biconvex sections. While these calculations are of a preliminary nature, they indicate a significant weakening of shock strength with decreasing aspect ratio. The point of greatest shock strength lies quite close to the tip, of the order of less than a chord for blade geometries considered here. The location of this point appears to be insensitive to tip Mach number and to depend on aspect ratio alone, being farthest from the tip for the largest aspect ratio.

A relatively simple formula is derived for subcritical flow conditions for which linearized theory is valid. This formula includes effects of compressibility, gives the correct first approximation to the three-dimensional flow near the tip and reduces to blade element theory inboard from the tip. Comparisons with more exact computer results show this asymptotic approximation in the linearized theory to be quite good up to tip Mach numbers of about 0.7.

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